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# Selective Multi-Carrier Index Keying OFDM: Error Propagation Rate with Moment Generating Function

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**Abstract**—We propose a new selective multi-carrier index keying in orthogonal frequency division multiplexing (OFDM) systems that opportunistically modulate both a small subset of sub-carriers and their indices. Particularly, we investigate the performance enhancement in two cases of error propagation sensitive and compromised device-to-device (D2D) communications. For the performance evaluation, we focus on analyzing the error propagation probability (EPP) introducing the exact and upper bound expressions on the detection error probability, in the presence of both imperfect and perfect detection of active multi-carrier indices. The average EPP results in closed-form are generalized for various fading distribution using the moment generating function, and our numerical results clearly show that the proposed approach is desirable for reliable and energy-efficient D2D applications.

**Index Terms**—Multi-carrier index keying, orthogonal frequency division multiplexing, error propagation.

## I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been employed in the majority of today and future communication standards such as IEEE 802.11, 3GPP's LTE-Advanced, due to its robustness to multipath fading. The performance of these systems with increased sub-carriers is heavily dependent on an increased sensitivity to mismatched conditions such as frequency offset and phase noise as well as transmission nonlinearity caused by the non-constant power ratio of OFDM symbols. Multi-carrier index keying (MCIK) has been recently proposed as a promising technique to improve the efficiency of traditional OFDM in [1], [2]. Similar to the spatial modulation (SM) concept, MCIK-OFDM (also, named OFDM index modulation) exploits the sub-carrier index as additional resource to decrease the detection errors over the classical OFDM at a low complexity with only a few sub-carrier activation. Due to its inherently

improved energy and spectral efficiency, MCIK-OFDM is a promising modulation scheme for providing the high data-rate especially for low-cost, energy constrained wireless systems such as device-to-device (D2D) communications.

Accurate and energy-efficient sub-carrier index detection is a key aspect of MCIK-OFDM systems. The effects of channel estimation errors on the approximate pairwise error probability (PEP) was discussed in [2], using the maximum likelihood (ML) detector. Recently, the tight bit error rate (BER) expression of MCIK-OFDM has been derived for any number of active sub-carriers in [3]. The impact of sub-carrier correlation on the detection error probability was investigated in [4], [5]. However, for the successful development of MCIK-OFDM D2D wireless systems, which is envisioned to consist of thousands of low-cost, energy constrained devices, low-complexity detection schemes are studied [6]–[8]. Moreover, to increase the reliability further, multiple antenna basis OFDM with index modulation was investigated in [9], [10].

In this work, we propose a method of selective MCIK-OFDM system that opportunistically activates partial clusters of sub-carriers based on partial channel information and modulate both a subset of sub-carriers and their indices, per activated clusters. Particularly, we consider two cases of wireless systems which are limited to or compromised by error propagation, and it is aimed to decrease the error probability of the proposed systems, taking into consideration a flexible number of active sub-carrier indices. Using moment generating function approach, a generalized error propagation performance has been analyzed for various fading distribution and corresponding factor of reducing the error propagation is theoretically examined, e.g., increasing the overall diversity order twice. The results clearly show that the proposed approach is vital for low-cost, energy-efficient wireless systems such as D2D applications.

## II. SYSTEM MODEL

### A. MCIK-OFDM

We consider a device-to-device MCIK-OFDM system with  $N_c$  sub-carriers that consists of  $n$  clusters of  $N$  sub-carriers (i.e.,  $N_c = nN$ ). Assume that each device is equipped with a single antenna, which will be extended to a multi-antenna setup in Section III-C.

In every transmission, only  $K$  out of  $N$  sub-carriers per cluster are randomly activated to deliver data symbols, while  $N-K$  sub-carriers are zero padded. A stream of M-QAM symbols is first serial-to-parallel converted, where every  $K$  ( $\leq N$ ) symbols are grouped into a vector  $\mathbf{s} = [s_1, s_2, \dots, s_K]^T$ ,  $s_i \in \mathcal{S}$ .  $\mathbf{s}$  are used to modulate sub-carriers, as in the classical OFDM, but it differs from that the modulated sub-carriers are only those of  $K$  activated indices. Hereinafter, we focus on a single cluster, otherwise stated. Details can be applied directly to other clusters, without loss of generality.

Denote by  $\mathcal{J}$  a set of the active indices for each cluster.  $\mathcal{J}$  is  $\mathcal{J} = \{i_1, \dots, i_K\}$  where  $i_\beta \in [1, \dots, N]$  for  $\beta = 1, \dots, K$ . For the  $K$  active indices per cluster, a different stream of  $m_0$  bits is used to randomly select  $K$  out of  $N$  sub-carriers and thus, only  $nK$  out of  $N_c$  sub-carriers are randomly activated in every transmission.

Taking into account both  $\mathcal{J}$  and  $\mathbf{s}$ , an MCIK-OFDM block is formed as  $\mathbf{x} = [s(1), \dots, s(N)]^T$  where  $s(k) \in \{0, \mathcal{S}\}, \forall k$ . Unlike the classical OFDM,  $\mathbf{x}$  in the proposed system comprises  $N-K$  zero elements whose indices help carry information of  $m_0$  bits.

In such MCIK-OFDM, the total number of active index combinations is  $B(N, K)$  but for the simplicity and effective mapping of the data bits,  $2^{\lfloor \log_2 B(N, K) \rfloor}$  combinations are used, where  $B(\cdot)$  and  $\lfloor \cdot \rfloor$  denote the binomial coefficient and the floor function, respectively. Therefore, in every MCIK-OFDM transmission,  $m_0 = \lfloor \log_2 B(N, K) \rfloor$  bits are used to modulate the sub-carrier indices by the MCIK and  $m_1 = K \log_2 M$  bits are transmitted by  $K$  symbols in  $\mathbf{s}$ .

The input-output model of MCIK-OFDM for each cluster can be given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \quad (1)$$

where  $\mathbf{y} = [y(1), \dots, y(N)]^T$ ,  $\mathbf{x}$  denotes an MCIK-OFDM signal block,  $\mathbf{H} = \text{diag}(h(1), \dots, h(N))$  where  $h(k)$  represents Rayleigh fading channel, being independent and identically distributed (i.i.d.) complex Gaussian with zero mean and unit variance, i.e.,  $h(k) \sim \mathcal{CN}(0, 1)$ , and  $\mathbf{n} = [n(1), \dots, n(N)]^T$  denotes the independent, additive white Gaussian noise (AWGN) vector, i.e.,  $n(k) \sim \mathcal{CN}(0, N_o), \forall k$ . We denote the signal-to-noise

ratio (SNR) by  $\rho = E_s/N_o$  where  $E_s$  denotes the average power for the M-QAM symbol.

For detection, we employ a two-step maximum likelihood (ML) detector: the classical ML detector only on  $s(k)$  based on  $y(k)$  is not sufficient to retrieve data bits in the proposed system. This is because the MCIK-OFDM conveys another bits by the random combination of active indices. Thus, we demand two decision processes: a likelihood ratio test (LRT) detects  $\mathcal{J}$  (and thus  $m_0$ ); and the ML detector retrieves  $m_1$  bits from the sub-carriers of the corrected and equalized active indices from the estimate  $\hat{\mathcal{J}}$ :

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2.$$

This decoder is optimal at the cost of the additional decoder of sub-carrier index (e.g., see [2], [3] for details).

### B. Selective MCIK-OFDM

We propose a new method of selective multi-cluster MCIK-OFDM, taking into account channel information. For a success MCIK-OFDM realisation, it is desired to primarily perform a reliable detection on the indices of the active sub-carriers that carry the data symbols. This is because an incorrect detection of the active indices causes propagation of errors in the detection of the data symbols. Thus, it is aimed to reduce error propagation probability (EPP), enhancing a success index detection.

Denote by  $\rho_k$  the receive SNR per sub-carrier  $k$ .  $\rho_k$  is defined as  $\rho_k = \phi \rho |h(k)|^2, \forall k$ , where  $\phi$  denotes the uniform power allocation coefficient to the  $K$  active sub-carriers, i.e.,  $\phi = N/K$ , such that the average power allocation per cluster is normalized.

For a success MCIK-OFDM, we propose a selective MCIK-OFDM system which is aware of an error propagation information per cluster. Particularly, it is intended to transmit  $\mathbf{x}$  if and only if  $\min_k \rho_k \geq \mu$ , for a given minimum desired SNR  $\mu$ . Corresponding MCIK-OFDM signal block can be written as

$$\mathbf{x} \mathcal{I}(\min_k \rho_k - \mu), \quad (2)$$

where  $\mathcal{I}(A) = 1$  if  $A > 0$ , otherwise zero. Therefore,  $\mathbf{x}$  per cluster is selected to make the on-and-off transmission, referring to  $\min_k \rho_k \geq \mu$ .

Especially, we focus on two cases:

- (i) Error propagation (EP) compromised MCIK-OFDM: Consider  $\mu = 0$  and thus, the selective MCIK-OFDM is reduced simply to the existing MCIK OFDM which is tolerant to the error propagation.
- (ii) Error propagation (EP) sensitive MCIK-OFDM: Consider a positive  $\mu \gg 0$ . As  $\mu$  increases, the

MCIK-OFDM for applications highly sensitive to error propagation can be designed. It aims to enhance the error propagation probability.

For this, it is assumed that the binary selection status,  $\mathcal{I}(\min_k \rho_k - \mu)$ , is known to each cluster via one feedback bit from the receiver.

### III. EP PERFORMANCE ANALYSIS

#### A. EP compromised case

1) *Exact instantaneous PEP and EP probability:* Denote by  $(\alpha \rightarrow \tilde{\alpha})$  the pairwise error event (PEE) that an active sub-carrier index  $\alpha$  is incorrectly detected as the index of the inactive sub-carrier  $\tilde{\alpha}$  for  $\alpha, \tilde{\alpha} \in \{1, \dots, N\}$  and  $\alpha \neq \tilde{\alpha}$ , given that  $\alpha$  is transmitted per cluster. Assume that  $\mathbf{H}$  is known to the receiver. Then, the conditional PEP can be defined as the probability of having the PEE,  $P(\alpha \rightarrow \tilde{\alpha})$ .

Using the ML detector, the well-known conditional PEP can be obtained as

$$P(\alpha \rightarrow \tilde{\alpha}) = Q\left(\sqrt{\frac{x_{\alpha, \tilde{\alpha}} \phi \rho}{4}}\right), \quad (3)$$

where  $x_{\alpha, \tilde{\alpha}}$  is the squared Euclidean distance  $x_{\alpha, \tilde{\alpha}} = |h(\alpha) - h(\tilde{\alpha})|^2$  and  $Q(x) \triangleq \pi^{-1} \int_0^{\pi/2} e^{-x^2/2 \sin^2 \theta} d\theta$  is the error function.

Notice that, for the MCIK-OFDM system with  $K$  out of  $N$  active sub-carriers, the error propagation probability (EPP) can be equivalent to the overall instantaneous PEP. The EPP takes into consideration the miss-detection of sub-carrier indices. Hence, using the law of the total probability and the union bound, the upper bound on the EPP can be obtained, in the EP-compromised case, as:

$$P_{ep-c} \leq \frac{K}{N} \sum_{\alpha=1}^N \sum_{\tilde{\alpha} \neq \alpha=1}^{N-K} P(\alpha \rightarrow \tilde{\alpha}). \quad (4)$$

Inserting (5) into (4), we obtain the EPP but it is not simple enough to see the impact of  $N$  and  $K$  on  $P_{ep-c}$ .

2) *Upper bounds on PEP and EP probability:* Using the bound on  $Q(x) \leq 0.5 \exp(-x^2/2)$  and after some simplification, (3) can be bounded as

$$P(\alpha \rightarrow \tilde{\alpha}) \leq 0.5 \exp(-\rho_{nm} x_{\alpha, \tilde{\alpha}}), \quad (5)$$

where we denote by  $\rho_{nm}$  the normalized SNR and  $\rho_{nm}$  is defined as  $\rho_{nm} = \phi \rho / 8$ . Accordingly, inserting (5) into (4), the EPP can be obtained as

$$P_{ep-c} \leq \frac{K}{N} \sum_{\alpha=1}^N \sum_{\tilde{\alpha} \neq \alpha=1}^{N-K} \frac{1}{2} \exp(-\rho_{nm} x_{\alpha, \tilde{\alpha}}). \quad (6)$$

It can be clearly seen from (6) that  $P_{ep-c}$  relies on the balance between  $K$  and  $N$ . It is worth pointing out that (6) is simpler than (4), enabling to theoretically look insight the EPP performance behavior.

3) *Average EP probability:* Denote by  $\bar{P}_{ep-c}$  the average EP probability (AEPP) in the EP-compromised case. In general, the AEPP can be obtained taking an expectation of (4) along with  $P(\alpha \rightarrow \tilde{\alpha})$  being either (3) or (5). Hence, we provide the following lemma.

*Lemma 1:* For the case of EP-compromised MCIK-OFDM, the AEPP can be expressed, with respect to the moment generating function, as

$$\begin{aligned} \bar{P}_{ep-c} &\leq \frac{K}{2N} \sum_{\alpha=1}^N \sum_{\tilde{\alpha} \neq \alpha=1}^{N-K} \mathcal{M}_{x_{\alpha, \tilde{\alpha}}} \{-\rho_{nm}\} \\ &= \frac{K(N-K)}{2} \mathcal{M}_{x_{\alpha, \tilde{\alpha}}} \{-\rho_{nm}\}, \end{aligned} \quad (7)$$

where  $\mathcal{M}_x(t)$  represents the moment generating function (m.g.f.) of the random variable  $x$ .

*Proof:* Please see the Appendix A. ■

It is clearly seen from (7) that the AEPP relies on the balance between  $K$  and  $N$ , and the normalized SNR  $\rho_{nm}$  ( $= \phi \rho / 8$ ) also affects the AEPP for given  $\phi = N/K$ . Notice that the AEPP in (7) is obtained while all clusters are always selected with no regard to the error propagation, i.e., threshold  $\mu = 0$ , and it is called the compromised error propagation.

#### B. EP sensitive case

Unlike the previous case, we now consider a positive value  $\mu > 0$  and select the acceptable clusters each holding  $\min_k \rho_k \geq \mu$ . Denote by  $P_{ep-s}$  the EPP in the EP sensitive case. Using  $\mathcal{I}(\cdot)$  in (2) and (5),  $P_{ep-s}$  can be represented by

$$P_{ep-s} \leq \frac{K}{2N} \sum_{\alpha=1}^N \sum_{\tilde{\alpha} \neq \alpha=1}^{N-K} \exp(-\rho_{nm} x_{\alpha, \tilde{\alpha}}) \mathcal{I}(x_{\alpha, \tilde{\alpha}} - 2\mu'), \quad (8)$$

where  $\mu' = \mu / (\phi \rho)$ , and  $\mathcal{I}(\cdot)$  returns one only when  $x_{\alpha, \tilde{\alpha}} \geq 2\mu'$  that is necessary to have both  $x_{\alpha} = |h(\alpha)|^2 \geq \mu'$  and  $x_{\tilde{\alpha}} = |h(\tilde{\alpha})|^2 \geq \mu'$ .

The corresponding the average EPP in this case can be obtained by taking the expectation of (8). The following lemma is provided.

*Lemma 2:* For the case of EP-sensitive MCIK-OFDM, the AEPP can be expressed as

$$\begin{aligned} \bar{P}_{ep-s} &\leq \frac{K}{2N} \sum_{\alpha=1}^N \sum_{\tilde{\alpha} \neq \alpha=1}^{N-K} \mathbb{E} \{e^{-\rho_{nm} x_{\alpha, \tilde{\alpha}}} \mathcal{I}(x_{\alpha, \tilde{\alpha}} - 2\mu')\} \\ &= \frac{K(N-K)}{2} \left( \mathcal{M}_{x_{\alpha, \tilde{\alpha}}}(-\rho_{nm}) \right. \\ &\quad \left. - \int_0^{\mu'} \int_{2\mu'}^{\infty} e^{-\rho_{nm} z} f_{x_{\alpha, \tilde{\alpha}}}(z) f_y(y) dz dy \right), \end{aligned} \quad (9)$$



where  $f_x(\cdot)$  represents the probability density function (p.d.f.) of the random variable  $x$  and  $y = \min_{\alpha} x_{\alpha}$ .

*Proof:* Please see the Appendix B. ■

The AEPP in (9) is clearly seen to rely on the balance between  $K$  and  $N$ , and is inversely proportional to the normalized minimum SNR  $\mu'$  for given  $\rho_{nm}$ . Intuitively, it describes that as  $\mu$  increases, the number of the EP events occurrence reduces, influencing the selection rate of the MCIK-OFDM. This leads to decreasing the AEPP at high  $\mu$ .

### C. Special distributions

As for distribution of  $\rho_k$ , consider two special distributions: exponential distribution; and Gamma distribution.

1) *Exponential distribution:* In our system model with a single antenna setup,  $x_{\alpha} = |h(\alpha)|^2$  follows the exponential distribution and accordingly,  $x_{\alpha, \tilde{\alpha}}$  used in  $\bar{P}_{ep-x}$  can be modeled as a chi-squared distribution with four degrees of freedom, i.e.,  $x_{\alpha, \tilde{\alpha}} = x_{\alpha} + x_{\tilde{\alpha}}$ , and its p.d.f is  $f_{x_{\alpha, \tilde{\alpha}}}(z) = ze^{-z}$ . In such a chi-squared distribution, therefore, it is well known that we can have

$$\mathcal{M}_{x_{\alpha, \tilde{\alpha}}}(t) = (1 - t)^{-2}. \quad (10)$$

**Theorem 1:** For the EP-compromised MCIK-OFDM, the AEPP can be given in closed-form as

$$\bar{P}_{ep-c} \leq \frac{K(N - K)}{2} (1 + \rho_{nm})^{-2}, \quad (11)$$

where recall that  $\rho_{nm} = \phi \rho / 8$  and  $\phi = N/K$  is the power allocation coefficient.

*Proof:* The proof is straightforward by using (10) and Lemma 1. ■

It can be shown from (11) that the AEPP of the error propagation compromised case solely relies on the balance between  $K$  and  $N$ , for given  $\rho$ .

**Theorem 2:** For the EP-sensitive MCIK-OFDM, the AEPP is written, in closed-form, as

$$\bar{P}_{ep-s} \leq \frac{K(N - K)}{2} \frac{\Gamma(2, N\mu' + (1 + \rho_{nm})2\mu')}{(1 + \rho_{nm})^2}, \quad (12)$$

where  $\Gamma(\cdot, \cdot)$  denote the upper incomplete gamma function.

*Proof:* For the EP-sensitive MCIK-OFDM, we can also derive the expression for the AEPP. To that end, using (10) and Lemma 2, the AEPP is represented as

$$\bar{P}_{ep-s} \leq \frac{K(N - K)}{2} \left( (1 + \rho_{nm})^{-2} - \int_0^{\mu'} \int_{2\mu'}^{\infty} z e^{-(1+\rho_{nm})z} N e^{-Ny} dz dy \right). \quad (13)$$

Employing the incomplete gamma function [11] and after some simplification, (13) can be reduced to (12). ■

It can be clearly observed from (12) that the AEPP in the EP-sensitive case decreases with a proper choice of  $K$  and  $N$ . Also, for given  $K$  and  $N$ , (12) shows that the AEPP decreases exponentially with the normalized minimum SNR  $\mu' = \mu/(\phi\rho)$ .

Moreover, it is worth pointing out from (11) and (12) that over the EP-compromised case, the AEPP in the EP-sensitive case decreases by a factor of  $\Delta_e$ :

$$\Delta_e = \Gamma(2, N\mu' + (1 + \rho_{nm})2\mu'),$$

and this factor significantly decreases as  $\rho_{nm}$  increase for a given  $\mu'$ .

2) *Gamma distribution:* We extend our system model to a multiple antenna setup. Let  $x_{\alpha}$  be modeled as a Gamma distributed random variable, i.e.,  $x_{\alpha} \sim \mathcal{G}(L, \theta)$ , where  $L$  represents the shape parameter (equivalent to the number of antennas) and  $\theta = 1$  the scale parameter. Accordingly,  $x_{\alpha, \tilde{\alpha}}$  used in  $\bar{P}_{ep-x}$  is also a Gamma distribution, i.e.,  $x_{\alpha, \tilde{\alpha}} \sim \mathcal{G}(2L, \theta)$ . The m.g.f of such a Gamma distribution is known as

$$\mathcal{M}_{x_{\alpha, \tilde{\alpha}}}(t) = (1 - t)^{-2L}. \quad (14)$$

**Theorem 3:** For the EP-compromised MCIK-OFDM over the Gamma distribution, the AEPP expression is given as

$$\bar{P}_{ep-c} \leq \frac{K(N - K)}{2} (1 + \rho_{nm})^{-2L}. \quad (15)$$

*Proof:* To prove (15), insert into Lemma 1 (14). ■

It can be found from (15) that the AEPP over the Gamma distribution relies on  $L$  as well as the balance between  $K$  and  $N$ . Particularly, as  $L$  increases, the AEPP in (15) is clearly seen to exponentially decrease, and the diversity order equals to  $2L$  in high SNR regime.

**Theorem 4:** For the EP-sensitive MCIK-OFDM over the Gamma distribution, the AEPP can be represented as

$$\bar{P}_{ep-s} \leq \frac{K(N - K)}{2(1 + \rho_{nm})^{2L}} \left( \frac{\Gamma(2L, (1 + \rho_{nm})2\mu')}{\Gamma(2L)} \times \frac{\Gamma(L, \mu')^N}{\Gamma(L)^N} \right). \quad (16)$$

*Proof:* The proof of (16) is similar to one in Theorem 2, except that the m.g.f. (14) and the p.d.f. of the gamma distribution are used. ■

It can be observed from (16) that similar to the exponential distribution, the AEPP decreases with a proper choice of  $K$  and  $N$ . Also, for given  $K$  and  $N$ , the AEPP exponentially decreases with  $\mu'$ , obtaining the diversity order of  $2L$ .

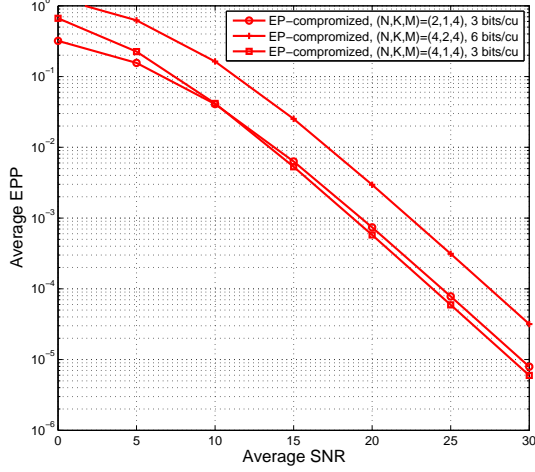


Fig. 1. Average EPP performance of the selective MCIK-OFDM for a D2D in the EP-compromised case over the Exponential distribution.

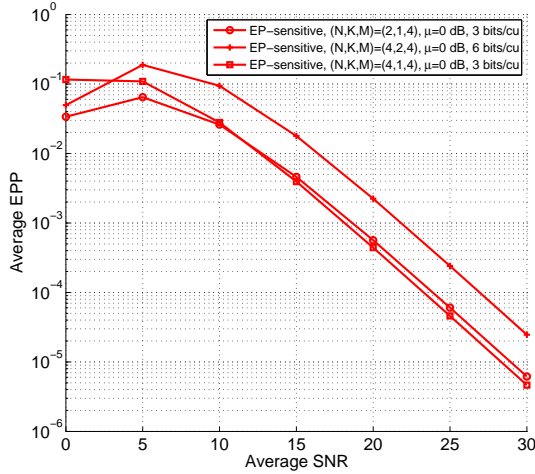


Fig. 2. Average EPP performance of the selective MCIK-OFDM for a D2D in the EP-sensitive case over the Exponential distribution.

Also notice that over the EP-compromised case, the AEPP in the EP-sensitive case decreases, in Gamma distribution, with a factor of  $\Delta_{\mathcal{G}}$ :

$$\Delta_{\mathcal{G}} = \left( \frac{\Gamma(2L, (1 + \rho_{nm})2\mu')}{\Gamma(2L)} \frac{\Gamma(L, \mu')^N}{\Gamma(L)^N} \right),$$

and this factor is influenced with  $\mu'$  and  $\rho_{nm}$ .

#### IV. NUMERICAL RESULTS AND DISCUSSIONS

We consider the selective MCIK-OFDM system with  $N_c = 128$  sub-carriers comprising of  $n = \{64, 32\}$  clusters of  $N = \{2, 4\}$  sub-carriers for various configurations of  $(N, K) = \{(2, 1), (4, 1), (4, 2)\}$ . The average EPPs are evaluated by referring to the upper bound expressions (7) and (9) for the EP compromised case and the EP sensitive case, respectively.

Fig. 1 depicts the average EPP of the selective MCIK-OFDM for the EP-compromised D2D system on the

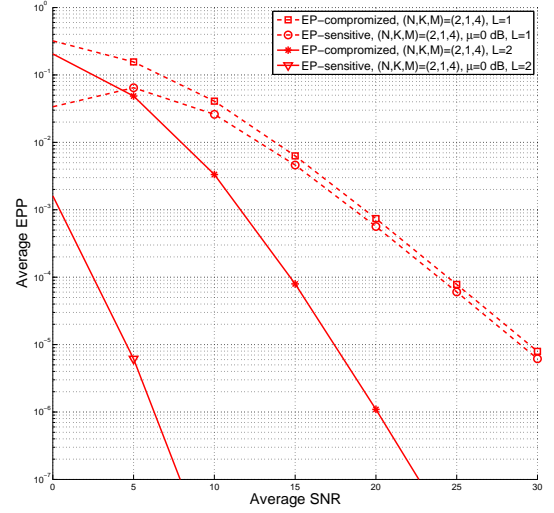


Fig. 3. Comparison of the EP-compromised and the EP-sensitive cases: Average EPP performance of the selective MCIK-OFDM for a D2D are depicted over the Exponential ( $L = 1$ ) and the Gamma ( $L = 2$ ) distributions.

Rayleigh flat fading per sub-carrier. For this, we consider the presence of both imperfect and perfect detection of active sub-carrier indices. This figure illustrates that the average EPP performs best with  $(N, K) = (4, 1)$  for a given  $M = 4$ . For small  $K/N = 1/4$ , intuitively, the OFDM transmission allocates the total power to smaller number  $K$  of active sub-carriers which can reduce the error propagation at the same data rate by the MCIK.

Fig. 2 illustrates the average EPP of the selective MCIK-OFDM suitable to the EP-sensitive D2D applications. For the illustrations, we use  $\mu = 0$  dB and  $M = 4$ . As seen in this figure, the average EPP at low SNRs (i.e.,  $\rho \leq \mu$ ) can be obtained low. Choosing a proper balance between  $K$  and  $N$ , such an EPP performance is shown to decrease further, which validates our theoretical analysis.

For comparison, Fig. 3 depicts the average EPP of both the EP-compromised and the EP-sensitive cases. Also, both the exponential distribution (i.e.,  $L = 1$ ) and the Gamma distribution (i.e.,  $L = 2$ ) are concerned for each case. As seen in this figure, the EP-sensitive case outperforms the EP-compromised one in terms of the average EPP, especially at low SNRs. This benefits from the opportunistic selection of the MCIK, leading to significant power gain at the cost of outage. It holds even when  $L$  increases (i.e.,  $L = 2$ ). Thus, the selective MCIK-OFDM is highly desirable for a better reliability of the low-power D2D, especially at low SNRs.

#### V. CONCLUSION

We proposed the selective MICK-OFDM scheme that opportunistically modulates both the sub-carriers and their indices in order to convey the information bits via

only a small subset of properly activated sub-carriers. To measure the performance, we derived the generalized error propagation probability expressions in closed-form taking into account the EP-compromised and the EP-sensitive cases, for various fading channel distribution. The derived average EPP expression will be useful to evaluate various concepts of the MCIK-OFDM for low-complexity, energy-efficient D2D applications.

## APPENDIX

### A. Proof of Lemma 1

Taking an expectation of the PEP  $P(\alpha \rightarrow \tilde{\alpha})$ ,  $\bar{P}_{ep-c}$  can be formulated as

$$\bar{P}_{ep-c} \leq \frac{K}{N} \sum_{\alpha=1}^N \sum_{\tilde{\alpha} \neq \alpha=1}^{N-K} \mathbb{E} \{P(\alpha \rightarrow \tilde{\alpha})\}, \quad (17)$$

where  $\mathbb{E}\{\cdot\}$  denotes the expectation operator.

Inserting (5) into (17), we have

$$\bar{P}_{ep-c} \leq \frac{K}{2N} \sum_{\alpha=1}^N \sum_{\tilde{\alpha} \neq \alpha=1}^{N-K} \mathbb{E} \{e^{-\rho_{nm}x}\}. \quad (18)$$

Notice that the outcome of the expectation in (18) gets independent of  $\alpha$  and  $\tilde{\alpha}$  and thus, (18) can be reduced as  $\bar{P}_{ep-c} \leq \frac{K(N-K)}{2} \mathbb{E} \{e^{-\rho_{nm}x}\}$ . Here,  $\mathbb{E} \{e^{-\rho_{nm}x}\}$  can be represented by the m.g.f of  $x$  and the proof of (7) is done.

### B. Proof of Lemma 2

Similar to (17),  $\bar{P}_{ep-s}$  can be formulated as

$$\bar{P}_{ep-s} \leq \frac{K}{N} \sum_{\alpha=1}^N \sum_{\tilde{\alpha} \neq \alpha=1}^{N-K} \mathbb{E} \{P(\alpha \rightarrow \tilde{\alpha}) \mathcal{I}(x_{\alpha, \tilde{\alpha}} - 2\mu')\}. \quad (19)$$

Using (5) and the p.d.f.  $f_{x_{\alpha, \tilde{\alpha}}}(\cdot)$ , (19) is written by

$$\begin{aligned} \bar{P}_{ep-s} &\leq \frac{K}{2N} \sum_{\alpha=1}^N \sum_{\tilde{\alpha} \neq \alpha=1}^{N-K} \int_{\mu'}^{\infty} \int_{2y}^{\infty} e^{-\rho_{nm}z} f_{x_{\alpha, \tilde{\alpha}}}(z) f_y(y) dz dy \\ &= \frac{K}{2N} \sum_{\alpha=1}^N \sum_{\tilde{\alpha} \neq \alpha=1}^{N-K} \left( \mathbb{E} \{e^{-\rho_{nm}z}\} \right. \\ &\quad \left. - \int_0^{\mu'} \int_{2\mu'}^{\infty} e^{-\rho_{nm}z} f_{x_{\alpha, \tilde{\alpha}}}(z) f_y(y) dz dy \right). \end{aligned} \quad (20)$$

Using the m.g.f. and after some simplification, the proof of (9) is done.

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